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TECHNICAL AND ECONOMIC STOCHASTIC MODELING OF NEW GENERATING FACILITIES TO MITIGATE UNCERTAINTY IMPACTING THE FEASIBILITY STUDY PROJECTIONS

The research relates to the challenges on improving the accuracy of simulating objects with inaccurate/uncertain technical and nontechnical parameters where different deterministic and stochastic modeling techniques have been developed to deal with uncertainties in decision making for renewable energy sources RES, where is exploiting the probabilistic methods like Monte Carlo Simulation and Point Estimate methods.

The three-point estimation technique was employed to analyze the stochastic parameters of the model which is represented by the distribution functions of their probable value.

A synthetic skewed probability density function (PDF) was modeled based on the standard normal distribution, which is fitting for modeling the uncertain parameters for renewable energy in power systems.

Key words: Three-point approximation simulation, stochastic parameters modeling, renewable energy sources in power systems, uncertainty modeling.

Introduction. Unlike the traditional power systems studies whose were mostly deterministic; the stochastic issue is being fundamental nowadays and the future [1], due to the growth of randomness phenomena following the growth on the integration of the RESs in power systems, there still certain challenges, these can be considered technical and/or nontechnical; as the latest is related to the capital cost, energy market, economical, and financial issues. moreover, the first challenge is related to the high level of variability in the among of energy sources (solar irradiation, wind speed) in the first class [2], wherein factors and indices are characterized by some uncertainty.

Three-point estimation techniques are a suitable tool for modeling contemporary energy systems based on the integration of RESs [3], As practical techniques of the three-point approximation are used to construct the PDF of the stochastic parameters model which directly influences the modeling result like an event occurrence probability.

Purpose and objectives of the research. Applications of The Three-Point Probability Density Function Approximation for modeling technical and economic parameters on the power systems based on RES, to demonstrate the possibility of modeling the stochastic input parameters of the simulated system using PDFs by the use of three-point approximation techniques and to obtain analytical expressions/results.

Research materials and results.

I. IMPACT OF LEVELIZED COST OF ELECTRICITY ESTIMATION

For known reasons, the Levelized Cost of Electricity (LCOE) is a measure of the generation cost of electricity, in constant currency, for a plant using a particular generating technology over its lifespan, wherein the major components of the LCOE equation are:

$$LCOE = \frac{Life \quad Cycle \quad Cost}{Life \quad Time \quad Energy \quad \mathrm{Production}}$$

Due to the increasing penetration level of RESs in power systems, more accurate predict values of the "LCOE" are demanded. Unlike , the simple estimation methods which may give less precise results of the uncertain parameter under study; wherein [4] shows the variance under these conditions can reach almost 50% the computed value of the variance for the LCOE and/or tariff price estimates for the energy produced by solar photovoltaic power plants (PVPP) and small PV-installations.

II. CONCEPTUAL BASIS OF THE THREE-POINT PDF APPROXIMATION USED

To demonstrate the possibility of improving the accurate results, in [6] a comparable numerical results of cost estimates and prices for electricity produced by PVPP are obtained using Monte Carlo Method and Point Estimation Method given the impact of uncertain parameters with the PDF function graphically depicted in Fig.1.

A) Distribution Functions Features

The simplest three-point PDFs approximation used are the *normal distribution*, *uniform*, and the distribution with the *triangular* PDF and their standard characteristics of common distribution functions are given in Table 1.

Table 1 Standard Characteristics of The Distribution Functions Used

Descriptive statistics	**Uniform**	***PERT***)* **(Beta-distribution)**	**Triangular**	**Normal**
Expected value (mean, or first moment M_1)	$\frac{A+C}{2}=$ $=\frac{2b+c-a}{2}$	$\frac{A+4b+C}{6}=$ $=\frac{6b+c-a}{6}$	$\frac{A+b+C}{3}=$ $=\frac{3b+c-a}{3}$	b
Dispersion, D	$\frac{(C-A)^2}{12}=$ $=\frac{(a+c)^2}{12}$	$\frac{(C-A)^2}{36}=$ $=\frac{(a+c)^2}{36}$	$(A^2+b^2+C^2-Ab-bC-AC)/18$	$\frac{(C-A)^2}{36}=$ $=\frac{(a+c)^2}{36}$
The standard deviation σ	$\frac{C-A}{12^{1/2}}=$ $=\frac{a+c}{12^{1/2}}$	$\frac{(C-A)^2}{6}=$ $=\frac{(a+c)^2}{6}$	$D^{1/2}$	$\frac{C-A}{12^{1/2}}=$ $=\frac{a+c}{12^{1/2}}$
Most preferable	does not exist	b	b	b

*)PERT – Program Evaluation and Review Technique.

The following notation of three points, which are characterizing the interval of possible values concerning the uncertain parameter:

b = x0 is the "most desirable" value of the uncertain parameter;

while $A = x_0 - a$ is the minimum or pessimistic value of the interval parameter (may also be "worst" or "best");

and $C = x_0 + c$ is the most expected or optimistic value of the parameter (in contrast to the previous ones, it will be "best" or "worst").

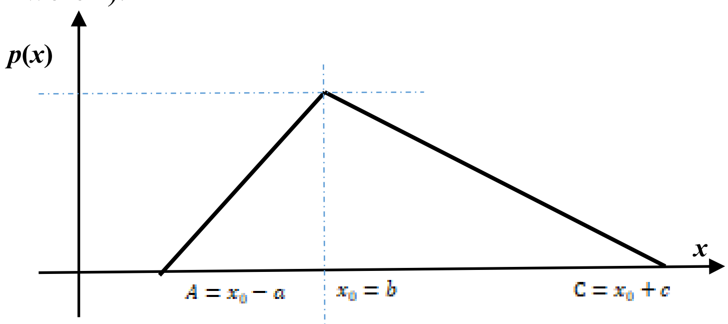

Figure 1 – General Diagram an asymmetric triangular PDF of uncertain parameter

Wherein, the triangular PDF is a proper tool for geometrical analysis, but it will not always represent the properties of the considered object adequately when the feasibility is to be studied. In [7] a proposition of a synthetic *Pseudo-normal* distribution PDF, which is asymmetrical to avoid the disadvantages inherent the PERT-Beta in estimating the time of service. This will alternatively serve like the Beta-PERT distribution function to represent uncertain parameters of the stochastic model.

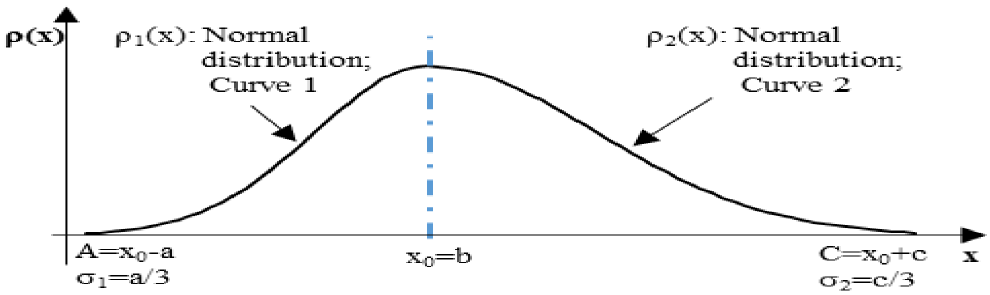

Figure 2 – The density of asymmetric normal probability distribution of the variable parameter

The synthetic curve of distribution in Figure 2 is formed by two shapes of the normal distribution curves with different values of the standard parameters M_1 and σ, where its probable density distribution function values are given in form of two fragments by the expression:

$$\rho(x) = \begin{cases} \rho_1(x) = Be^{-\frac{(x-x_0)^2}{2\sigma_1^2}} & \forall x \in [x_0 - 3\sigma_1, x_0] \\ \rho_2(x) = Be^{-\frac{(x-x_0)^2}{2\sigma_2^2}} & \forall x \in [x_0, x_0 + 3\sigma_2] \end{cases}$$

$$with: \int_{-\infty}^{\infty} \rho(x)dx = 1 \tag{1}$$

$$and: B = \frac{2}{\sqrt{2\pi}} \frac{\rho_{norm}}{(\sigma_1 + \sigma_2)}.$$

Thus, for random values ξ of the parameter x, the values of the integral probability distribution function Cumulative Density Function (CDF) can be computed with the obtained expression (3) for computer computing:

$$\int_{x_0-3\sigma_1}^{x_0} \rho_1(x)dx + \int_{x_0}^{x_0+3\sigma_2} \rho_2(x)dx = -\Phi^*(-3) + \Phi^*(3) = 2\Phi^*(3) - 1 \tag{2}$$

$\Phi^*(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{\xi^2}{2}} d\xi$ – is a Laplace function with known properties, and $\Phi^*(-3) = 1 - \Phi^*(3)$.

And after interchanging, we got:

$$P_2(\xi \le x) = \frac{2\sigma_1 \times \rho_{norm}}{(\sigma_1 + \sigma_2)} \left(\Phi^*(3) - \frac{1}{2} \right) + \frac{2\sigma_2}{(\sigma_1 + \sigma_2)} \left(\Phi^* \left(\frac{x - x_0}{\sigma_2} \right) - \frac{1}{2} \right). \tag{3}$$

In Figure 3, we presented graphical curves of the approximating functions of the variable parameter distributions computed by the use of the three-point estimation techniques. Figure 3 (a) – is positive, and Figure 3 (b) - is negative symmetry of distribution curves of some parameter with mode value b = 15. Figure 4 – shows the curves of these distributions for the case of the symmetric shape of the distribution of values of such a variable parameter of the model.

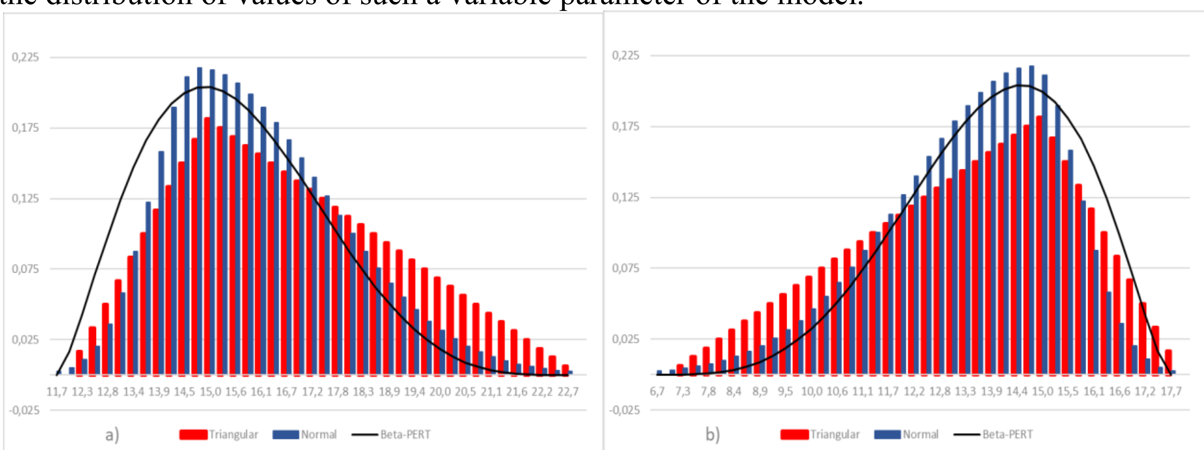

Figure 3 – Left & Right skewed PDFs: a) postive skewness (A = 12, b = 15, C = 23); b) postive skewness (A = 7, b = 15, C = 18)

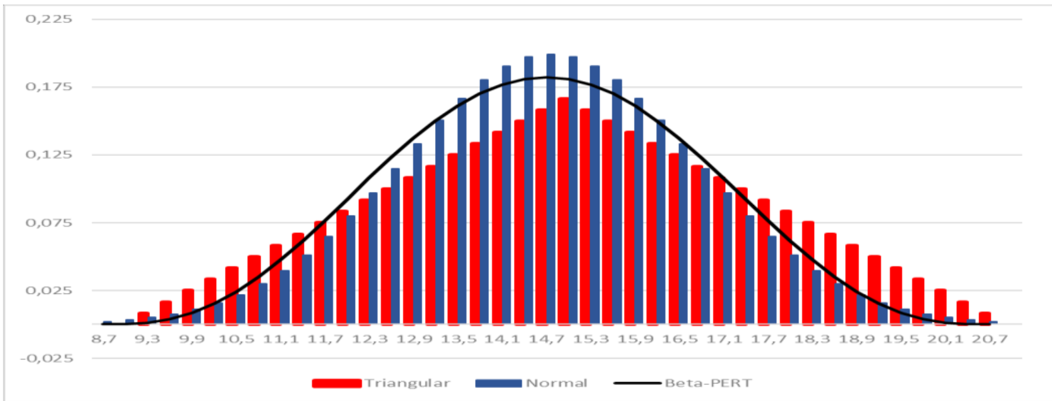

Figure 4 – Symmetric PDF curves: (a = c = 6; A = 9, b = 15, C = 21)

Practical Example: Model Information Support in Research [7 – 9] is based on data on the engineering and economic performance of the PV installation, The PDF used in Figure 5 is highly similar to the asymmetric PERT distribution with the PDF, given the sampling interval of uncertain System Degradation Rate (SDR) value limited by 3%, and the desired value is of 0.8% (the mode). The expected SDR value of the mode for practical computations, applicable for newly constructed objects in Ukraine, equals 1.2% (Fig. 5), while the sampling interval accepted to be within 4% and the approximated synthetic normal PDF function is used.

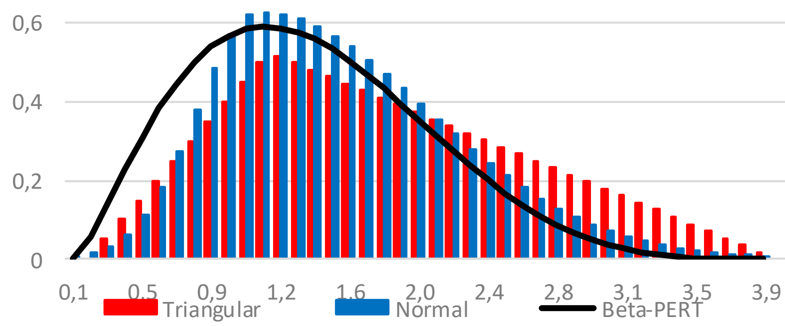

Figure 5 –Comparison of the approximation distribution functions curves for PV installation model "system degradation rate,%/year"

III. COMPUTATION OF THE INITIAL MOMENTS OF THE ASYMMETRIC NORMAL DISTRIBUTION FUNCTION

Algebraic expressions were presented to compute the Moments with the order -s- of a synthetic approximating function and determined by integrating:

$$M_s(x) = \int_{-\infty}^{\infty} x^s \rho(x)dx = \int_{-\infty}^{x_0} x^s \rho_1(x)dx + \int_{x_0}^{\infty} x^s \rho_2(x)dx = \\ = B\left[\int_{x_0-3\sigma_1}^{x_0} x^s e^{-\frac{(x-x_0)^2}{2\sigma_1^2}}dx + \int_{x_0}^{x_0+3\sigma_2} x^s e^{-\frac{(x-x_0)^2}{2\sigma_1^2}}dx\right], \forall s = 1,2,3,... \tag{4}$$

The first moment M_1 (mathematical expectation) of function (1):

$$\begin{aligned} M_1(x) &= \int_{-\infty}^{\infty} x\rho(x)dx = \int_{-\infty}^{x_0} x\rho_1(x)dx + \int_{x_0}^{\infty} x\rho_2(x)dx = \\ &= B\left[\int_{x_0-3\sigma_1}^{x_0} xe^{-\frac{(x-x_0)^2}{2\sigma_1^2}}dx + \int_{x_0}^{x_0+3\sigma_2} xe^{-\frac{(x-x_0)^2}{2\sigma_2^2}}dx\right] = \\ &= B\int_{b-a}^{b} xe^{-\frac{(x-b)^2}{2\sigma_1^2}}dx + B\int_{b}^{b+c} xe^{-\frac{(x-b)^2}{2\sigma_2^2}}dx \end{aligned} \tag{5}$$

The integrals in (5) can be determined by applying the traditional replacement of the integration variables:

$\frac{x-b}{\sigma_1} = t, dx = \sigma_1 dt, x = \sigma_1 \times t + b$ – for the first integral in (5).

$\frac{x-b}{\sigma_2} = t, dx = \sigma_2 dt, x = \sigma_2 \times t + b$ – for the second integral in (5).

$$\int_0^c e^{-\frac{t^2}{2}}dt = \sqrt{2\pi}(\Phi^*(3) - 0.5) \approx \sqrt{2\pi}/2$$

$$\int_{-a}^0 e^{-\frac{t^2}{2}}dt = \sqrt{2\pi}(\Phi^*(3) - 0.5) \approx \sqrt{2\pi}/2$$

Finally, given the approximation acceptable for practical computations $\Phi^*(3) \approx 1$, and therefore p_{norm} =1

$$M_1(x) = B\left[\frac{b\sqrt{2\pi}}{2}(\sigma_1+\sigma_2) + \sigma_2^2 - \sigma_1^2 + \sigma_1^2 e^{-\frac{a^2}{2}} - \sigma_2^2 e^{-\frac{c^2}{2}}\right] \qquad B = \frac{2}{\sqrt{2\pi}}\frac{1}{(\sigma_1+\sigma_2)} \tag{6}$$

Using the same scheme, we consistently define expressions to compute the second and third initial moments in the form:

$$M_2(x) = B\left[\frac{\sqrt{2\pi}}{2}\left[b^2(\sigma_1+\sigma_2) + \sigma_1^3 + \sigma_2^3\right] + 2b\sigma_1^2\left(e^{-\frac{a^2}{2}} - 1\right) - a\sigma_1^3 e^{-\frac{a^2}{2}} + 2b\sigma_2^2\left(1 - e^{-\frac{c^2}{2}}\right) - c\sigma_2^3 e^{-\frac{c^2}{2}}\right] \tag{7}$$

$$M_3(x) = B\left[\begin{aligned} &\frac{b\sqrt{2\pi}}{2}\left[\sigma_1\left(b^2+3\sigma_1^2\right) + \sigma_2\left(b^2+3\sigma_2^2\right)\right] + \sigma_1^2\left(3b^2+2\sigma_1^2\right)\left(e^{-\frac{a^2}{2}} - 1\right) + \\ &+\sigma_2^2\left(3b^2+2\sigma_2^2\right)\left(1-e^{-\frac{c^2}{2}}\right) + a\sigma_1^3(a\sigma_1 - 3b)e^{-\frac{a^2}{2}} - c\sigma_2^3(c\sigma_2+3b)e^{-\frac{c^2}{2}} \end{aligned}\right] \tag{8}$$

By applying the "3σ – rule" in terms of the notations a, b, and c in Figure 2 – and substituting, into formulas (6), (7) and (8), we obtain expressions for practical computations.

Though extra effort to interpret the results with variance analysis is usually needed, the analytical expressions (1)–(3) of the CDF integral probability function were used for simulation using the Monte Carlo simulation scheme in [8,9].

Concerning the expressions of Moments obtained, they would help to speed up computations based on the Point Estimation method to obtain the uncertain parameter value of the Deterministic and Stochastic Model set by PDF three-point approximation technique developed herein.

Conclusion. For the reason of improving the simulation results accuracy in the problems of estimating the technical and economic indicators for energy objects, the author proposed an approximate function of a synthetic normal distribution with asymmetry, analytical expressions for computing the characteristics which are suitable for the construction of deterministic and stochastic.

Combined with three-point approximation techniques for variable parameters and Point

Estimation Method (method of Moments) regarding D&SM, it gives significant computing resources savings while processing the numerical solutions of practical economic and mathematical problems.

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